2.3-2.5 Derivative Rules: Form A Key

Tutor copy. Every answer was checked by symbolic computation from the formulas as printed (SymPy, `verify_form.py`). Provenance: `PROVENANCE_FORMA.md`. To the extent possible under law, the author(s) have waived all copyright and related or neighboring rights to this key. CC0 1.0 Universal: `https://creativecommons.org/publicdomain/zero/1.0/`. Prepared with assistance from Claude, an AI model made by Anthropic.

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General grading rules

- **Show work.** A bare correct answer earns at most half credit.
- **Any valid method.** Expanding first, negative exponents, the quotient rule, simplifying first, solving for y first where possible, or carrying the rules all the way through are all accepted; no method is required.
- **Notation.** y', $\dfrac{dy}{dx}$, $f'(x)$ and $\dfrac{d}{dx}[\ ]$ are all accepted.
- **Value vs. derivative.** A function value where a derivative was asked (or a first derivative where a second was asked) earns 0 for that part.
- **Simplifying.** Answers need not be simplified; equivalent forms are correct.
- **Excluded values.** Where a formula fails at some values, the key may name them. No points are deducted for omitting statements of excluded values.
- **Trap notes** print the line that a skipped step leaves out, next to the answer the skip produces.

Question 3 columns run x, $g(x)$, $f''(x)$, $f(x)$, $g'(x)$, $f'(x)$.

Form A

Item	Answer	Work, rubric, trap
1(a)	$-2x\csc x$ $-(3-x^2)\csc x\cot x$	Product rule: $(-2x)\csc x+(3-x^2)(-\csc x\cot x)$. *Rubric:* 1 pt each product-rule term; 1 pt both signs. *Trap:* $\dfrac{d}{dx}[\csc x] = -\csc x\cot x$. Writing $+(3-x^2)\csc x\cot x$ drops that minus; the line to write is $(3-x^2)\cdot(-\csc x\cot x)$.
1(b)	$\dfrac{-2x^2-2x-8}{(x^2-4)^2}$	$\dfrac{2(x^2-4)-(2x+1)(2x)}{(x^2-4)^2} = \dfrac{2x^2-8-4x^2-2x}{(x^2-4)^2}$. *Rubric:* 2 pts quotient rule in the right order; 1 pt answer (unsimplified is fine). *Trap:* The minus must reach both terms of $(2x+1)(2x) = 4x^2+2x$: the line to write is $-(4x^2+2x) = -4x^2-2x$. $-4x^2+2x$ is the slip.
1(c)	0	No x appears: $y = \frac{1}{2}\cdot 1$, a constant. *Rubric:* 3 pts, all or nothing. *Trap:* The product rule here gives $\cos(\frac{\pi}{6})\cot(\frac{\pi}{4})+\ldots$, treating fixed angles as x.
1(d)	$36x^{-4}+\frac{3}{4}x^{-1/2}$	$y = 6x^{-2}+x^{3/2}$; $y' = -12x^{-3}+\frac{3}{2}x^{1/2}$; $y'' = 36x^{-4}+\frac{3}{4}x^{-1/2}$. *Rubric:* 1 pt rewriting with exponents; 1 pt y'; 1 pt y''. *Trap:* $(-12)(-3) = +36$; $-36x^{-4}$ is the dropped sign. Stopping at y' answers a different question (0 pts).
2(a)	$\sin x$	Every four derivatives return $\cos x$; $35 = 4\cdot 8+3$, so the answer is the third derivative: $-\sin x$, $-\cos x$, $\sin x$. *Rubric:* 2 pts the cycle of four; 1 pt $35 = 32+3$; 2 pts answer. *Trap:* $-\sin x$ is the first derivative, not the 35th. The 35 counts derivatives; $\cos^{35} x$ would be a power.

Item	Answer	Work, rubric, trap
2(b)	18 (at $t = 3$)	$v = 6t^2 - 18t = 6t(t-3) = 0$ at $t = 0, 3$; only $t = 3$ has $t > 0$. $a = 12t - 18$; $a(3) = 18$. *Rubric:* 1 pt v; 1 pt $t = 3$ (excluding $t = 0$); 1 pt a; 2 pts 18. *Trap:* $a(0) = -18$ is excluded by $t > 0$. $t = 3$ and $s(3) = -22$ are not accelerations.
3(a)	$-\frac{5}{2}$	$q' = \dfrac{g'f - gf'}{f^2} = \dfrac{(1)(2) - (-4)(-3)}{2^2} = \dfrac{2 - 12}{4}$. *Rubric:* 2 pts quotient rule with g on top; 1 pt reads; 2 pts value. *Trap:* Reversing the numerator, $\dfrac{gf' - g'f}{f^2}$, gives $+\frac{5}{2}$. $(-4)(-3) = +12$, so the line is $2 - 12$.
3(b)	-3	$p' = 2xf + x^2f'$; $p'' = 2f + 2xf' + 2xf' + x^2f'' = 2f + 4xf' + x^2f''$; at 1: $4 - 12 + 5$. *Rubric:* 2 pts p'' formula; 1 pt reads; 2 pts value. *Trap:* $2 \cdot f''(1) = 10$ differentiates each factor twice and multiplies. Missing the second $2xf'$ gives $4 - 6 + 5 = 3$.
4(a)	$-10x\csc^2(5x^2)$	$-\csc^2(5x^2) \cdot 10x$. *Rubric:* 1 pt outer $-\csc^2$; 1 pt inner $10x$; 1 pt answer. *Trap:* Dropping the inner $10x$, or the minus of $\dfrac{d}{dx}[\cot x]$.
4(b)	$-\frac{8}{3}(1-4x)^{-1/3}$	$f = (1-4x)^{2/3}$; $f' = \frac{2}{3}(1-4x)^{-1/3} \cdot (-4)$. *Rubric:* 1 pt exponent $\frac{2}{3}$; 1 pt inner -4; 1 pt answer. *Trap:* Exponent $\frac{3}{2}$ (root and power swapped); losing the inner -4 gives a positive answer.
4(c)	$-6\cos^2(2t)\sin(2t)$	$\cos^3(2t) = (\cos(2t))^3$: $3\cos^2(2t) \cdot (-\sin(2t)) \cdot 2$. *Rubric:* 1 pt power layer; 1 pt $-\sin(2t)$; 1 pt inner 2. *Trap:* $\cos^3(2t)$ is a cube of a cosine, not $\cos(6t)$ or $\cos((2t)^3)$.
4(d)	$\sec(x^2)$ $+2x^2\sec(x^2)\tan(x^2)$	Product: $1 \cdot \sec(x^2) + x \cdot \sec(x^2)\tan(x^2) \cdot 2x$. *Rubric:* 1 pt product rule; 1 pt sec tan; 1 pt inner $2x$. *Trap:* $\sec(x^2)\tan(x^2) \cdot 2x$ alone skips the product rule.
5(a)	2	$g(-3) = 3$, $g'(-3) = -1$; $f'(3) = -2$ (segment from $(2,3)$ to $(4,-1)$); $k'(-3) = (-2)(-1)$. *Rubric:* 1 pt $g(-3)$, $g'(-3)$; 2 pts f' at 3; 1 pt value. *Trap:* $f'(-3)\,g'(-3) = (1)(-1) = -1$ reads f' at -3; the sign flips.
5(b)	-3	$f(-2) = 3$, $f'(-2) = 1$; $g'(3) = -3$ (segment from $(2,2)$ to $(4,-4)$); $m'(-2) = (-3)(1)$. *Rubric:* 1 pt $f(-2)$, $f'(-2)$; 2 pts g' at 3; 1 pt value. *Trap:* $g'(-2)\,f'(-2) = (-1)(1) = -1$ reads g' at -2.
6	-25	$f(2) = (-1)^3 = -1$; $f'(x) = 6x(x^2-5)^2$, $f'(2) = 12$; $y = -1 + 12(x-2)$; at $x = 0$: $-1 - 24$. *Rubric:* 1 pt $f(2)$; 2 pts $f'(2)$; 1 pt line; 2 pts -25. *Trap:* The equation of the line is not the y-intercept; $f(0) = -125$ is the y-intercept of f. $-1 + 24 = 23$ drops the sign of $12(0-2)$.
7(a)	$5(x^3 - \sin x)^4$	With $s = x^3 - \sin x$ the expression is $\dfrac{d}{ds}[s^5] = 5s^4$. *Rubric:* 3 pts, all or nothing. *Trap:* Differentiating with respect to x gives $5(x^3 - \sin x)^4(3x^2 - \cos x)$ (0 pts).
7(b)	0	$\tan^2 u - \sec^2 u = -1$. The chain route: $6\tan(3x)\sec^2(3x) - 6\sec^2(3x)\tan(3x) = 0$. *Rubric:* 3 pts, all or nothing for 0; either route. *Trap:* Leaving $6\tan\sec^2 - 6\sec^2\tan$ uncombined is correct but students often stop believing it is 0.
8(a)	$\dfrac{3 - 2xy}{x^2 + \csc^2 y}$	$2xy + x^2y' + \csc^2(y)\,y' = 3$; $y'(x^2 + \csc^2 y) = 3 - 2xy$. *Rubric:* 1 pt product rule on x^2y; 1 pt $+\csc^2(y)y'$; 1 pt right side 3; 1 pt solving. *Trap:* $-\dfrac{d}{dx}[\cot y] = -(-\csc^2 y\,y') = +\csc^2 y\,y'$: two minus signs. $x^2 - \csc^2 y$ in the denominator is the slip.
8(b)	$\dfrac{2y}{y - 2x}$	Expand: $4xy = y^2 + 8$, so $4y + 4xy' = 2yy'$ and $y'(4x - 2y) = -4y$. The direct route also works. *Rubric:* 1 pt expansion or chain rule on both squares; 1 pt product rule on xy; 1 pt $2yy'$; 1 pt solving. *Trap:* $-(x-y)^2 = -x^2 + 2xy - y^2$: the line to write distributes the minus to all three terms; the slip $-x^2 - 2xy - y^2$ cancels every term on the left and leaves $0 = y^2 + 8$.

Item	Answer	Work, rubric, trap
9(a)	1	$g'(y)\,y' + y + xy' = 0$; $y = 3$, so $g'(3) = -5$: $-5y' + 3 + 2y' = 0$, $y' = 1$. *Rubric:* 1 pt $g'(y)y'$; 1 pt product rule on xy; 1 pt read at $y = 3$; 2 pts value. *Trap:* g is applied to y; row $x = 2$ gives $-\frac{3}{1+2} = -1$, the opposite sign. Check: $g(3) + 2 \cdot 3 = 10$.
9(b)	$y - 3 = x - 2$	$y - 3 = 1 \cdot (x - 2)$. *Rubric:* 4 pts; follow-through from (a). *Trap:* Using $g(2) = -3$ or $g(3) = 4$ as a y-coordinate.
10	$y-2 = -\frac{4}{5}(x-1)$, $y-2 = 4(x+3)$	$x^2 + 2x + 4 = 7$: $x = 1$ or $x = -3$. $y' = -\dfrac{2x+y}{x+2y}$: $-\frac{4}{5}$ at $(1, 2)$, 4 at $(-3, 2)$. *Rubric:* 2 pts both points; 2 pts y'; 1 pt each slope; 1 pt both lines. *Trap:* $y = 2$ is a y-value: substituting $x = 2$ finds other points. At $(-3, 2)$ the line is $y - 2 = 4(x - (-3)) = 4(x + 3)$; $(x - 3)$ is the slip.
11	$\dfrac{2y+1}{\sec x \tan x}$	$\sec x \tan x \dfrac{dx}{dy} = 2y + 1$. *Rubric:* 2 pts differentiating with respect to y; 2 pts answer. *Trap:* $\dfrac{dy}{dx} = \dfrac{\sec x \tan x}{2y+1}$ is the reciprocal (0 pts).
12	$-\frac{5}{4}$	$y' = -\dfrac{x}{4y} = -1$; $y'' = -\dfrac{4y - 4xy'}{16y^2} = -\dfrac{4 - 16(-1)}{16} = -\dfrac{20}{16}$. *Rubric:* 2 pts y'; 4 pts quotient rule with y' inside; 2 pts value. *Trap:* Substituting $(4, 1)$ into y' first gives -1, whose derivative is 0. $-4xy' = -16(-1) = +16$; losing that sign gives $\frac{3}{4}$.

Points: 100.